

Contributions of cognitive neuroscience for learning fractions¹

Aportes de la neurociencia cognitiva para el aprendizaje de las fracciones

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Abstract

Investigating the relationship between Neurocognition and interlearning of fractions has reached greater depth and importance, due to the findings obtained in the educational field by Cognitive Neuroscience, which investigates the evolution of the mind, the brain and its plasticity. This research is based on 78 articles, from the last 34 years, recovered from the WoS, Scopus and Scielo databases. The research serves as the basis for understanding the contributions of neurocognition to interlearning of fractional numbers.

key words: neurocognition, interlearning, fractions

Resumen

Investigar la relación entre Neurocognición e interaprendizaje de fracciones ha alcanzado mayor profundidad e importancia, debido a los hallazgos obtenidos en el campo educativo por parte de la Neurociencia Cognitiva, que investiga la evolución de la mente, el cerebro y su plasticidad. La presente investigación está fundamentada en 78 artículos, de los últimos 34 años, recuperados de las bases de datos WoS, Scopus y Scielo. La investigación sirve de base para comprender los aportes de la neurocognición al interaprendizaje de los números fraccionarios.

Palabras clave: neurocognición, interaprendizaje, fracciones

1. Introduction

The educational transformation of the nineties incorporated a leading role for the student, making him an active agent in the inter-learning process; innovation that sought to develop in the student the learning to know, learning to do and learning to be, through the elaboration of a multidimensional curriculum, with the purpose of attending to the diversity of the learners, so as not to continue homogeneously teaching all student groups; educational paradigm that desired to combine quality with equity (Lieberman & Miller, 1999; Tesdesco & Tenti-Fanfani, 2002). Murillo in 2008, investigated the performance of basic subjects worldwide, the study determined that the performance of the students was at the unsatisfactory level despite considering the educational differences in the inter-learning processes in each country. This difficulty that students have to learn, could be

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aimed at teachers not including in their pedagogical activities the relevant contributions of Cognitive Neuroscience (Dekker et al., 2012).

In the inter-learning process, the Constructivist Theory, is the one of greater application within the classroom, because it makes possible in its practice the management of active methodologies oriented so that the student is a protagonist in the learning process, despite this, Constructivism does not consider the process of learning from the student at the neuronal level, so it does not develop activities that are parallel to the mental processes of the students in relation to learning. In this sense, research describes the need to include the findings of Cognitive Neuroscience, about how the brain works in the different inter-learning activities in the educational field (Pickering & Howard-Jones, 2007).

In the learning of Mathematics, one of the contents with greater difficulty to learn, are the fractions; the learning of rational numbers is the first approximation that students have to understand fractions (Gómez et al., 2014), although several alternatives are used to teach fractions there are students who do not achieve understanding their meaning and usefulness, causing that the students considered averages do not get to obtain a conceptual understanding when operating them (Fazio & Siegler, 2011), problematic that is a challenge for the basic education curriculum; on the basis that learning fractions is a necessary tool for the development of the student's multiplicative, quantitative and mathematical thinking (Thompson & Saldanha, 2003).

In 2007, the European Economic Organization for Development OECD, encourages first world countries to develop research in the Neurocognitive field, to benefit the inter-learning process ; supporting research centers to investigate how traditional teaching methods intervene in the inter-learning process within the classroom (Istance, 2008).

Cognitive Neuroscience continues to conduct research that allows us to see how memory, perception, reasoning and other brain processes interrelate in learning. Other questions are: how is the regulation of social consciousness presented in the brain and vice versa? And how do these transform neural networks and nuclei in the brain? (De la Barrera & Danilo, 2009; Fischer 2009; Goswami 2015; Puebla & Talma, 2011). For this reason, in the field of Mathematics, teachers must have the responsibility of developing cognitive skills in students so that they can compare, assess and order fractions, allowing the student a clear appropriation of the sense of magnitude and / or exact amount of a fraction (Capilla, 2016).

These reasons led the researchers to provide information on the findings of Cognitive Neuroscience for the benefit of the inter-learning process of fractions, scientific information that will provide people immersed in education a knowledge of how the brain works when learning fractions and what neurocognitive strategies the teacher can incorporate in the curricular design, teaching and evaluation of learning. Providing in this way a solution proposal to children, young people and adults who experience difficulties.

1.1. Developing

Humans and other animal species such as mammals, birds and amphibians have the capacity of sense of numerosness, which allows them to quantify the elements of the mediate environment and distinguish between enough and little (Serra-Grabulosa et al., 2010). When talking about numerical representation, natural numbers are represented in the human brain as a number line, where the numbers are spaced apart. Currently Cognitive Neuroscience provides sustainable bases to understand how the brain works in the inter-learning processes, some sciences dedicated to education incorporated these findings in curriculum design, teaching and learning evaluation (Puebla & Talma, 2011).

At present there is the proposal of the neurocognitive model that incorporates the triad (brain-mind-behavior) and its interaction and interrelation within a social learning environment, which aims to improve the efficiency

and effectiveness of the inter-learning that the students co-build in the classroom. To make the brain-mind-behavior, interact within the inter-learning process in the classroom, from a neurocognitive and Social Sciences perspective, consider two necessary principles (Howard – Jones, 2011; Howard – Jones, 2011):

a) Brain plasticity. It refers to the brain being perennially prepared for lifelong learning, provided with the ability to adapt to new circumstances, adjustment that develops specific changes in the brain's own structure, specifying that the brain is a sensitive, moldable and flexible to the experiences experienced in the environment, allowing perfect learning.

b) Learning as a social construction. It explains that the constant evolution of the brain is due to the interrelationships and interactions that people build in their links with other people and/or living beings. Human beings are exposed to socio-emotional-cultural influences, coming from their social coexistence; interaction that develops the construction of their identity, sense of belonging and value of sociocultural roots.

1.2. Neural bases of numerical processing

When a person works in a numerical representation, a specific area of his brain is activated and also his anterior and posterior zones; If a person locates two numbers close to each other and performs a comparison process, it will cause the overlapping effect, which will increase as the magnitude of the numbers increases. To determine the brain areas involved in the numerical processing was used: functional magnetic resonance, evoked potentials, magneto encephalography, positron emission tomography, among others, the application of these imaging techniques allowed us to propose new hypotheses about the learning process of Mathematics (Macizo et al., 2016). When working with numbers, several brain structures participate, of which 4 of great relevance were identified:

1. In the horizontal segment of the intraparietal sulcus it was observed through the fMRI that the (SHSIP) is notoriously activated when a person performs numerical processing tasks, than when performing different tasks. In addition, it was observed that this zone presents greater activation when the person is comparing two numbers in an estimated way, that when a calculation is made, at the same time this area is very important when ordering non-numerical ordinal series, such as alphabetic ones.

2. The angular gyrus presents great activity in calculation tasks, at the same time this area has high activity in verbal and reading tasks, therefore, it interacts with calculation tasks that have verbal components, such as multiplication. In addition, in this area, unlike (SHSIP), the activity is greater when the task is accurate and implies a calculation, so it decreases significantly when the task is approximation (Serra-Grabulosa et al., 2010).

3. In the frontal lobe, specifically in the prefrontal cortex brain activity is observed, when a person performs arithmetic calculation, plans and temporarily orders components for checking and correcting errors, however, greater brain activity occurs when the responses of the tasks are incorrect, specifically in the left lateral prefrontal cortex and middle and lower frontal gyrus (Muller & Knight, 2006).

4. In the Cingulate cortex, cerebral activity is observed when performing simple and complex arithmetic calculations, this area is considered as support because it does not have a specific function in numerical processing, its participation is evidenced in attention, working memory, decision making , monitoring and selection of responses (Allman et al., 2001).

Other research reveals that when learning Mathematics the participation of the parietal lobe is decisive to solve arithmetic problems and more precisely the intervention of the horizontal segment of the intra-parietal sulcus; because one of its main functions is: the internal representation of quantity, the abstract processing of the quantities and the resulting relationship between them. On the other hand, the angular gyrus takes part in the

verbal process of explicit arithmetic tasks and in solving sums of small quantities. Other studies reveal the involvement of the prefrontal cortex, the posterior part of the temporal lobe and various subcortical regions of the brain (Alonso & Fuentes, 2001; Butterworth, 1999; Blackmore & Frith, 2011; Dehaene, 1997; Dehaene et al., 1999; Dehaene et al., 2003; Gallese & Lakoff, 2005; Radford & André, 2009; Ballestra et al., 2006).

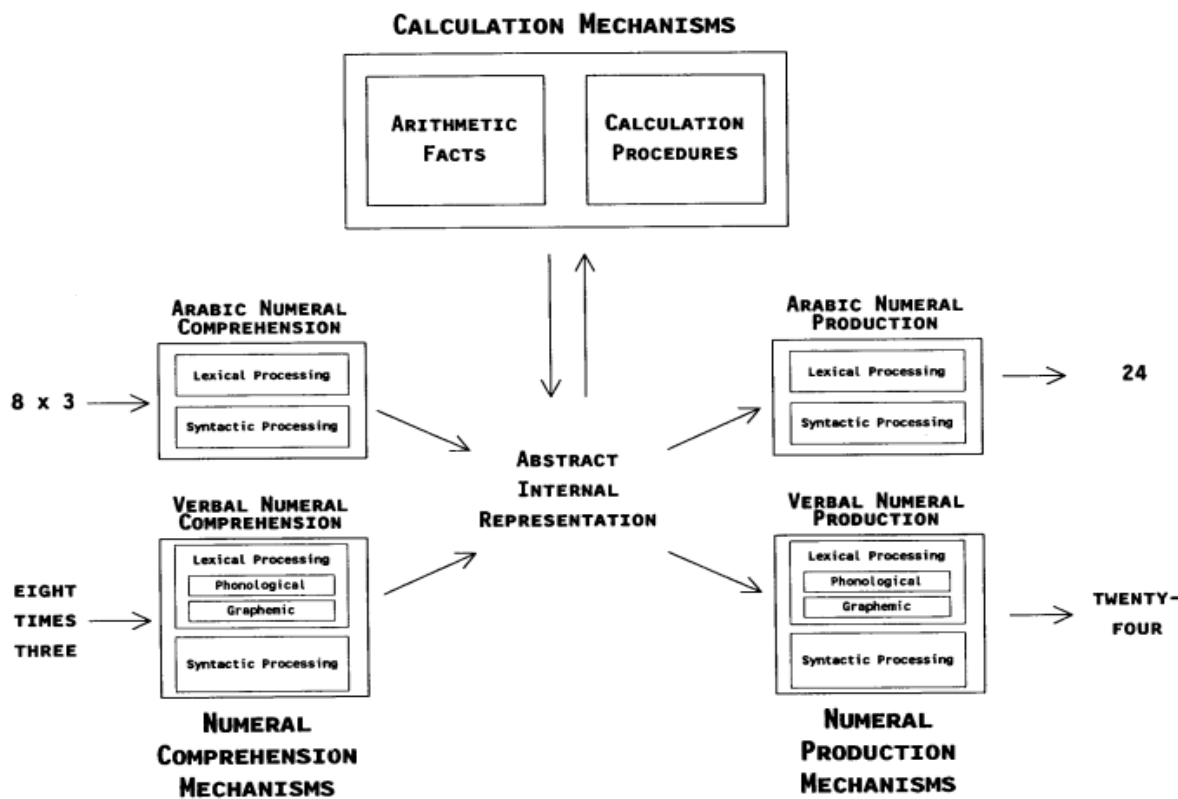
Previous studies propose explanatory theoretical models on numerical processing and calculation, which consider the development of numerical processing in the brain; There are three models of greater dissemination and acceptance: 1) the Cognitive Model proposed by McCloskey in 1992, 2) the Cipolotti & Butterworth Multi-Route Model in 1995 and 3) the Triple Code model formulated by Dehaene & Cohen in 1995, the latter being the most popular and accepted when researching the subject (Serra- Grabulosa et al., 2010; Serra-Grabulosa, 2013).

1.3. Cognitive models

1.3.1. Model of the numerical processing of McCloskey (1992)

Based on the difficulties to understand the mathematical concepts and symbols, the one that distinguishes differentiated components for the understanding and production of Arabic numerals and words, the existence of two specific cognitive systems that participate in the numerical processing is postulated, the first is the one that runs in a mechanism to understand and the other to produce numbers; adds the existence of different devices to process Arabic numerals and numerical words, the components that make it possible to understand an Arabic or verbal number, transform the inputs into an abstract internal representation that is then used in mathematical cognitive processes; production processes convert the internal representation of numbers into a numerical or verbal output (McCloskey et al., 1985; McCloskey, 1992; Orrantia & Muñoz, 2012).

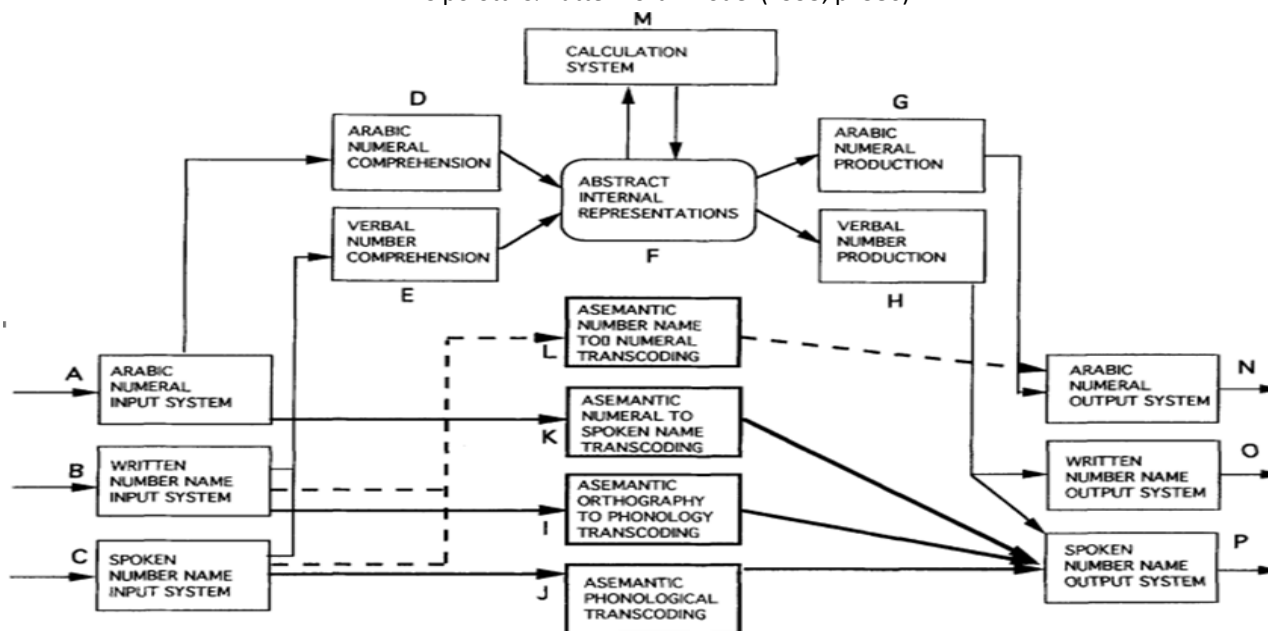
Figure 1
Model of numerical cognition and transcoding
by proposed (McCloskey 1992, p.113)



1.3.2. Multi-route model of Cipolotti & Butterworth (1995)

It proposes the existence of a system of abstract representations that is used when performing calculations. The model would be an extension of the one proposed by McCloskey, adding asemantic routes that allow conversion between some codes and others, without accessing the semantic representation of the number; the use of one route or another depends on the task, thus inhibiting the other routes.

Figure 2
Cipolotti & Butterworth model (1995, p. 386)



1.3.3. Model of the Triple Code of Dehaene (1992)

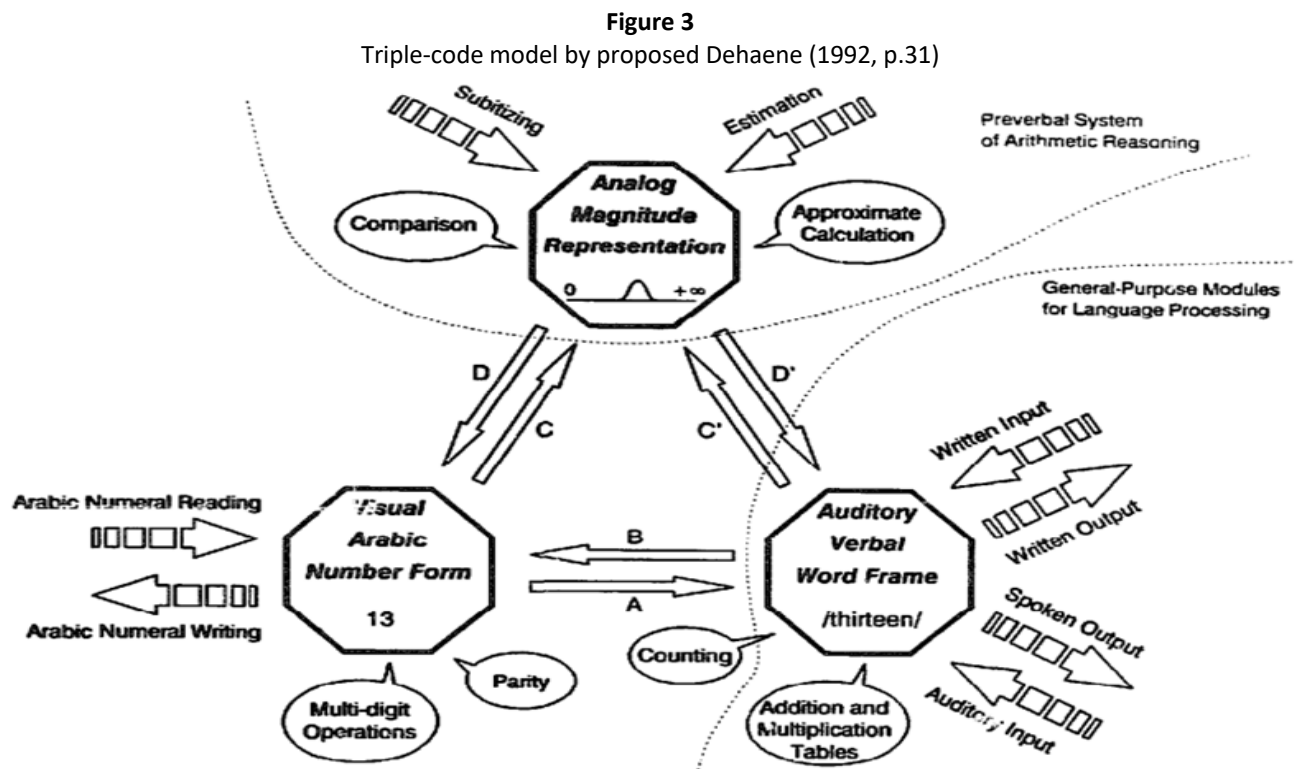
This model formulates that the information passes from one code to another through asemantic routes, the choice of the code will be according to the type of mental operation that is going to be performed, Dehaene expresses that if you want to learn comparison of magnitudes, the code that is would be the Analog, however, if you want to memorize the multiplication tables would correspond to the auditory Verbal code and if you would like to perform arithmetic calculations the code to be used would be the Visual (Serra-Grabulosa et al., 2010).

This Model transcends because it conceptualizes the numerical sense as the ability to represent continuous quantities, which are classified into analog and approximate representations, that is, they are mental operations that are not necessarily carried out with Arabic numerals. When the concepts are related to the digits, the development of the numerical sense in the brain-mind begins. This neuro-functional model postulates three hypotheses that are consistent with the neural site where the nodules that encode, decode and coordinate with the different tasks to be performed are located (Dehaene, 1997; Dehaene & Cohen, 1995); These hypotheses are:

1. The numerical information is managed from three codes: a) From an analog representation of quantity, in which the numbers are symbolized as a number line, a process that drives the right and left parietal areas of the brain. b) From a verbal-auditory format, where numbers are represented as strings of letters forming words, for example: when pronouncing thirty-seven, the perisylvian areas of the left hemisphere are activated and c) From an Arabic visual representation where the numbers are they symbolize as a string of digits, example; 13 245, a process in which the lower occipito-temporal zones of both hemispheres of the brain participate.

2. The transcoding procedures allow the information to be translated directly from one code to another, that is, depending on the task they are passed automatically through the asemantic routes.

3. Each calculation procedure is based on the existence of a consolidated link between the entry and exit of the codes. The mental processes that can be performed are: a) Comparison of magnitudes, b) Multiplications and simple sums, c) Subtraction and d) Multidigit operations.



Neuropsychological investigations allow to relate the neuroanatomic tentative circuits for each node of the neurofunctional model. It is reflected that the occipito-temporal ventral sectors of both hemispheres are involved in recognizing the form of the Arabic number, for example: the number 1; and in the left perisylvian areas that are related to the verbal representations of number, for example, write the One and the most important thing is that in the intra-parietal zone of both hemispheres is involved in the representation of analog quantities (Göbel et al., 2001; Serra-Grabulosa et al., 2010).

1.4. The analog representation of the number

A spelling is analog when the representation of a quantity changes over a continuous range of values, also because there is an intrinsic similarity between the phenomenon to be symbolized and the mental representation; guiding the existence of two systems of representations: 1) A structural one related to perceptions of an abstract type, where global forms and the structure of objects that build invariant representations on size and reflection are symbolized and b) The episodic representation codifies these transformations in memory (Ballesteros, 1993).

When graphing a magnitude, the numerical quantities are symbolized as a distribution of numbers on an analog number line, routed from left to right or vice versa depending on the culture; It is where the meaning of the number is represented, since neither the Arabic numeral form nor the verbal structure of the word contains semantic information. At this level, the quantity or magnitude associated with an explicit number, for example,

(3), is recovered and is related to numerical quantities, which is preferably used for rounding and for other numerical approximation and estimation tasks (Jacubovich, 2006).

The representation of an Arabic number, for example (6) becomes a visual representation that in the encoding and decoding processes is used first to perform calculation operations of one or more written digits and that require precision, generating two routes for the reading of numbers, a superficial semantic route that is based on the application of the corresponding conversion algorithms and according to the determined norms of the language, allowing to read any Arabic number even if it is the first time it is seen and a deep semantics route that only acts with familiar items that have a specific lexical input.

The findings obtained, orient to think that the reading of numbers is structurally similar to reading words, however, these two processes are functionally and neuroanatomically grounded in different routes, involving three main routes that are: the non-lexical surface, the deep semantics and the asemic lexicon (Salguero, 2007). The superficial route is based on the correspondence between letters and sounds, based on the grapheme-phoneme conversion rules (Cohen et al., 1994). The findings mentioned in relation to the numerical processing support the hypothesis that the intra-parietal sulcus and the horizontal segment are specifically responsible for the internal representation of the quantities and the abstract processing of the magnitudes, without differentiating the symbolic or non-symbolic format of the stimulus (Dominguez, 2008).

1.5. Rational numbers and their cognitive learning process.

The cognitive processes that the student must have developed before beginning the learning of rational numbers are: understand what an integer represents and the arithmetic operations that can be performed; besides its application in mechanical arithmetic exercises and in solving problems, you should know and use the hierarchy of arithmetic properties such as the commutative, associative, distributive and neutral element, the concepts of neutral, opposite and inverse elements, divisibility of numbers natural, Maximum Common Divider (GCF) and Minimum Common Multiple (MCM) (Cortina, 2014; Cortina et al., 2013).

The next step in the development of the inter-learning of rational numbers, is to know that the a/b symbolism acquires a restricted meaning when viewed as a division, this explained in another way would be that the fraction $2/5$ symbolizes dividing a whole into five similar parts and take two. When you want to explain the concept of fractional number, the most used examples in the educational field are fruits, pizzas, cakes and geometric figures, which ends up reducing the idea that truly involves the concept (Butto, 2013).

The inter-learning of the fractions introduces a succession of properties different from that of natural numbers, for its understanding it is necessary to differentiate that the fractions and the numerosness of a rational number manages to be represented through other rational numbers and that the fractions decrease when multiply and increase by dividing them. Understanding this activity would induce students to rethink their conceptual understanding of these numbers (Stelzer et al., 2016).

Another difficulty is that it is taught in the same way to operate fractions as when operating with whole numbers, without taking advantage of the capabilities of the Relationship Processing System (RPS). This reallocation can have serious organizational consequences (Ni & Zhou, 2005; Siegler & Pyke 2013), facilitating the deep reorganization of numerical reasoning develops the acquisition of the concept of fraction (Siegler & Schneider, 2011; Siegler & Pyke, 2013), when comparing the calculation-based method that relates thinking in terms of whole numbers, in relation to the appropriate articulation developed by the RPS, can help students expand the clear understanding of the arithmetic properties of fractional numbers (Matthews et al., 2016a; Lewis et al., 2016b; Matthews & Lewis, 2017).

The above points out the importance of learning rational numbers and the operations that can be done with them; it is highlighted that a large number of students, whether they are in basic, middle, high school and even higher education, experience difficulty in learning rational numbers and within this one that of fractions (Vamvakossi & Vosniadou, 2010). Research carried out by Cognitive Psychology in conjunction with the Education Sciences, identified some common difficulties that students have regarding the learning of rational numbers, which can be synthesized in three:

1. The difficulty of accessing the holistic symbolic magnitudes of rational numbers is because the description of the magnitude of a rational number is an important piece of conceptual knowledge that predicts accuracy in the calculation of fractions (Van Dooren et al., 2015; Siegler & Pyke, 2013).
2. The confusion between the concept of integer and rational number, which arises when talking about magnitudes, to understand the stipulated the following example is formulated: the integer has its own value that is indicated by the number, that is, five is worth five (5) and it is not difficult to say five (5) is greater than four (4) and less than six (6), however, in a rational number saying $7/3$ complicates recognizing if it is greater than $9/5$ or that if it is less than $6/2$, this bias develops because students often confuse the concepts and appropriate procedures to operate integers but inappropriate for fractions (Ni & Zhou, 2005).
3. The inability to examine that the properties of integers are different from that of rational numbers (Pecharromán, 2013; Vamvakossi & Vosniadou, 2010), these properties are: a) Rational numbers have no direct successor unlike integers, b) There is an infinite amount from one rational number to another. c) Multiplication by a positive rational number sometimes produces a result that is smaller than the initial one, while when divided it can produce a larger result, depending on the magnitude of the multiplier or divisor.

Cognitive Neuroscience guides Pedagogy through its findings from the student's relationship to learn to learn and unlearn to learn, thus emerging neurodidactics, which seeks to develop proposals for a meaningful inter-learning based on research that determines how the brain works in educational contexts of learning (Cuesta, 2009). The importance attributed to neurodidactics is that it guides the development of stimuli to increase the neural connections necessary for the brain of the students, making inter-learning possible, since it seeks to develop an interactive process between teacher – student.

According to neurological research, cognitive and brain processes are interrelated with didactics and neurology, if they are developed simultaneously and collaboratively, it is possible that effective strategies are generated from the teacher to the students (Cuesta, 2009). Findings in neurobiology confirm that previous experiences are transcendental for the development of a significant inter-learning (Saavedra, 2001).

One of the relevant contributions of Cognitive Neuroscience to the educational area, for the inter-learning process, is to demonstrate the ability of the central nervous system to modify itself through cerebral plasticity and how adequate early stimulation benefits this process (Carlson, 1996; Campos, 2014). In the educational field, specifically in the teaching of fractions, the teacher must guide the process of synaptic pruning, preventing students from developing biases (Pascual-Leone, 2001), which arise when learning to operate with rational numbers and/or fractional, learning that occurs around 12 years of age, stage in which the brain is in full development, so the use of varied teaching strategies will favor the process of brain plasticity necessary in students.

In non-radical contrast with what has been expressed about the presence of biases that prevent or hinder the learning of fractions, empirical evidence arises that proposes that the human being has a cognitive architecture that possibly has suitable systems to support this process of inter-learning, which means that there are neural networks capable of developing the learning of fractions (Piazza, 2010; Dehaene, 2011).

The spelling of the whole number is an ideogram, possible to discriminate and understand conceptually, developing a relatively easy learning for most students, due to the existence of a neurocognitive correlation of numerical representation systems with a long phylogenetic history (Dehaene & Cohen, 2007; Piazza, 2010). Lewis et al. (2015), the mistakes in understanding fractions do not arise from the discrepancy between the concept of fraction and the cognitive architecture of the student, but that the current methods of inter-learning are not viable, by wasting neurocognitive abilities and perceptive of the students. The authors formulate that teaching methods do not consider the following points:

1. The participation of the parietal lobe is decisive to solve arithmetic problems, the horizontal segment of the intraparietal sulcus is precisely involved, because its main specializations are the internal representation of quantity, the abstract processing of the magnitudes and their interrelation.
2. The angular gyrus is involved in the verbal process of explicit arithmetic tasks such as learning multiplication tables and the sums of small quantities, admits the resolution of mathematical problems. Other studies indicate the participation of the prefrontal cortex, the posterior part of the temporal lobe, the cingulate cortex and different subcortical regions (Serra-Grabulosa et al., 2010).
3. The Triple Code model allows access to abstract representations in three ways: verbal, visual and analog of magnitude. Studies allow us to relate the neuroanatomic tentative circuits of each of the nodes of the model. These possible areas are: the occipito-temporal ventral sectors of both hemispheres that are involved in recognizing the Arabic numeral form, the left perisylvian areas are involved in the verbal representation of the numbers and most importantly, it is that the intra-parietal zone of both hemispheres is involved in the representation of the analog quantity (Göbel et al., 2001; Serra-Grabulosa et al., 2010).
4. The findings in relation to numerical processing contribute to the hypothesis that the intra-parietal sulcus and the horizontal segment are specifically responsible for the internal representation of quantity and the abstract process of the magnitudes, without differentiating the symbolic or non-symbolic format of the stimuli (Dominguez, 2008).
5. The significant learning of fractions in basic education allows us to predict an advantage in the learning of Algebra and general Mathematics knowledge during higher basic education (Siegler et al., 2012). Developing that learning is relatively easy for most students, this is due to the existence of a neurocognitive correlation of representation systems (Dehaene & Cohen 2007; Piazza, 2010), despite the fact that this structure is found in some animal species and is not characteristic only of human beings (Jacob & Nieder, 2009).

1.6. Limitations of cognitive architecture

Research reveals a series of neurocognitive architectures called the Relationship Processing System (RPS), which are adapted to the magnitudes of the mono-symbolic and holistic coefficient. These investigations propose that the ability to represent the relationship between rational numbers and their magnitudes conferred by the (RPS) can support the understanding of fractions as a relative magnitude and even develop the conceptual thinking that rational numbers represent (Matthews et al., 2016a; Lewis et al., 2016b; Matthews & Lewis, 2017).

The predictions of (RPS) are:

1. The functioning of the neural circuits related to the learning of non-symbolic arithmetic operations predict individual differences, however in the symbolic fraction, the process is similar to the sharpness of the (ANS) which allows to predict mathematical achievements (Halberda et al., 2008; Libertus et al., 2011), however, even though the sharpness of the (ANS) correlates with Mathematics, it does so with less intensity than when working with the skills of a symbolic number (Chu et al., 2013; Lyons et al., 2014). Which suggests that the

representation of symbolic connections strengthens the links between (ANS) and symbolic representations, a process that predicts Mathematical ability.

2. One of the reasons why students cannot generate strong links between symbolic and non-symbolic processes, such as representations of the magnitude of the fraction, is because most of the teaching programs of fractions bring with frequency fractionation activities, which do not allow to activate the (ANS). One way to correct this situation is to develop perceptual learning to help the student associate the fractions with the magnitudes they represent (Lewis et al., 2016b).
3. At the neuronal level they can be integrated into the circuits of (RPS), neurophysiological mechanisms that participate in the learning of fractions. Research suggests that in the frontal parietal area there are cortical networks involved in the representation of the non-symbolic proportions, they are also involved in the processing and symbolic representation of the fractions, in addition patterns that are observed during the performance of sum operations and subtraction of fractions are activated (Schmithorst & Brown, 2004). Even the simple comparison of symbolic fractions activates a frontal parietal network and its dependent areas, maintaining an activation distance of the correct IPS area (Ischebeck et al., 2009).
4. Repeated presentation of symbolic fractions (Arabic numerals and pronunciation of the fraction in words) leads to adapt and solidify learning, which depends on the process of recovery of the frontal and parietal regions (Jacob & Nieder, 2009), which suggests that these networks are recruited, even in the absence of specific tasks.

1.7. Development line and the architecture of the RPS

There are no conclusive findings that support the development of the parietal cortex in relation to (ANS) and with respect to the ontogenesis of (RPS), the only neuroscientific evidence, is that of non-symbolic relationships, information that was obtained when studying a trained adults and monkeys (Vallentin & Nieder 2008; Izard et al., 2008). Outlining the brain's ability to represent non-symbolic relationships before, during and after a process of inter-learning of fractions, is a critical step not only in understanding how these neuronal architectures are prepared for students to learn symbolic fractions, but also in illuminating how these architectures can be shaped by formal education.

The component that analyzes the dimensions such as physical size and numerosness (IPS), are observed in regions near the areas associated with the non-symbolic proportion processing, increasing the possibility that the (RPS), becomes an extension of a higher order of one or more of these representation systems (Kadosh et al., 2005; Pinel et al., 2004).

Determining how these systems interact can help clarify the different principles of student learning when incorporating knowledge of continuous relationships and discrete relationships, can also help to warn about the educational debate about the relative effectiveness of different pedagogical representations for the fractions inter-learning process, which already has a long history (Cramer & Wyberg, 2009).

2. Methodology

To carry out this investigation, the databases of the Wos, Scopus and Scielo were reviewed, from 1985 to 2019; the selected articles had to meet the following requirements: 1) Studies in English, Portuguese and Spanish; 2) With publication date between April 1985 to August 2019; 3) the subjects studied could be of any ethnicity and sex and not only limited to children; 4) That the studies have a relationship between Cognitive Neuroscience and learning, Neuroscience and education. Studies aimed at patients with specific pathologies were excluded, the

study selection strategy consisted of an exhaustive literature search in the aforementioned databases and a manual search based on the references of the selected articles to locate studies not found in the previous search.

The inclusion criteria to select the studies were all those that refer to: brain functions and learning, neuroimaging and brain functions, Neuroscience and Mathematics learning; Neuroscience and Education, and Cognitive Psychology and fractional learning. 120 articles were selected in a preliminary way, articles that did not meet the inclusion criteria were eliminated, both through the reading of the title and its summary, as well as the complete critical reading of potentially relevant works. A total of 78 articles were selected that met the inclusion criteria, the selected articles were assessed for their methodological quality through the recommendations of the STROBE criteria with a qualitative evaluation (yes / no) carried out by the Authors independently.

3. Discussion

Currently, to know the areas that participate in cognitive processes in a specific way, neuroimaging techniques are used, which determined that in the numerical processing and calculation the parietal lobe is involved and specifically the intra-parietal sulcus for arithmetic problem solving; an addition, the angular gyrus intervenes in the verbal process of explicit arithmetic tasks and allows the resolution of mathematical problems. Other studies indicate the involvement of the prefrontal cortex, the posterior part of the temporal lobe, the cingulate cortex and different subcortical regions.

When the learning of rational numbers is well explained in basic education, it predicts an advantage in learning Algebra and Mathematics during higher basic education. Researchers of the subject express that many of the biases in the process of learning the fractions do not arise from the difference between the concepts of rational numbers and the cognitive architecture of the student, but rather arouse because the current methods of inter-learning do not use the contributions of cognitive neuroscience about how the brain learns when it exercises and reasons about the topic of fractions.

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